Fault Tolerant Control Strategy Based on the DoA: Application to UAV

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Abstract: In this paper, a fault tolerant control (FTC) strategy for an unstable Unmanned Aerial Vehicle (UAV) subject to control surface failures is presented. The aircraft has few redundancies and the actuator saturations are considered. The latter reduce the ability for the UAV to manoeuvre and this fact is emphasized in faulty mode. The FTC strategy aims at increasing the stability by enlarging the domain of attraction (DOA) while setting the dynamic of the closed-loop system. Considerations about the implementation of this FTC system are tackled and a discussion about the time allowed for diagnosis and reconfiguration is led.

Keywords: fault tolerant systems, saturation control, aircraft control, optimization problem, stability analysis.

1. INTRODUCTION

The studies dealing with faulty systems should take into account their actuator deflection ranges and the limits of their physical domain. As regards the systems with actuator faults, the efficiency of the control laws depends on healthy actuator saturation levels. It is all the more true that these systems are open-loop unstable since the actuator saturation levels determine partially the stable state space region (henceforth named domain of attraction or DOA). Yet, when an actuator fault occurs, the state vector moves away from its operating point with the risk to leave the DOA or the physical domain.

On the one hand, considering the healthy actuator saturation levels, it should be interesting to know the size of the DOA in faulty mode with the nominal controller. On the other hand, an FTC strategy should be designed to increase this region by considering the healthy actuator saturation levels. Therefore, two problems have to be considered:

(1) an analysis problem which consists in estimating the DOA of the system with its nominal controller in fault-free and in faulty mode.
(2) a synthesis problem which consists in designing a fault tolerant controller. Among other performances, this design should consider the DOA with respect to the healthy actuator saturation levels.

Various topics linked to the problems mentioned above have been studied for the purpose of the FTC. First, some fault tolerant control strategies take into consideration the input and output bounds. This is e.g. the case with predictive control (Maciejowski and Jones [2003]), (Chen [2006]). Other reallocation strategies based on optimization methods deal also with saturation levels (Zhang et al. [2007]), (Joosten et al. [2007]). However, among these methods, none is concerned with the DOA. In (Ito et al. [2002]), Ito addresses the analysis problem for a reentry vehicle with actuator saturations. The domain of stability is computed with an Algebraic Riccati Equation (ARE) approach. This analysis examines the stability domain in the event of a control surface actuator failure. In (Bateman et al. [2006]), the authors propose a combined analytic and simulation-based approach for assessing the stability of a control law for a F18 jetfighter that may be subject to actuator saturation due to failures and subsequent reconfiguration. Their analysis is based on a LMI approach which provides an estimation of the DOA. This estimation is used to guide the simulations in order to certificate complex nonlinear and reconfigurable control laws.

Analysis problem and synthesis of controllers for linear systems with actuator saturations have been studied in (da Silva Jr et al. [2002]), (Hu and Lin [2001]). In particular the domain of attraction obtained in (Hu and Lin [2001]) is less conservative than that provided by other available techniques. The goal of this paper is to apply some theoretical results published in (Hu and Lin [2001]) to fault tolerant control methods. Furthermore, an extension to these works is proposed to design an FTC controller which maximizes the DOA while the poles of the closed-loop system are placed into a region LMI.

The system used to illustrate these concepts is a UAV. The faults considered are stuck control surfaces. The paper is organized as follows. A short description of the UAV is given in section 2. In section 3, an estimation of the DOA of the closed-loop aircraft is computed. A fault
tolerant controller is designed in section 4, this design aims at increasing the DOA while the dynamic of the closed-loop is set. The simulation results are shown in section 5. Conclusions and considerations about the time required to diagnose and accommodate the faults are given in section 6.

2. UAV DESCRIPTION

The aircraft studied and shown on figure 1 is the Aerosonde UAV for which the Aerosim MATLAB toolbox was developed. Note that its inverted V-tail is common to many UAVs. Faults considered are asymmetric stuck control surfaces. In this perspective, the UAV model has been adapted to take account of the aerodynamic effects produced by each control surface, especially those which are on the boom mounted inverted V tail. The latter produce pitch and yaw moments, thus faults on these controls are particularly critical, all the more that other controls offer few redundancies. The controls are shown on figure 1: $\delta_2$ is the throttle, $\delta_{ar}$, $\delta_{al}$, $\delta_{fr}$, $\delta_{fl}$, $\delta_{el}$, $\delta_{el}$ control respectively the right and left ailerons, the right and left flaps and the right and left inverted V tail control surfaces.

$U = (\delta_2 \delta_{ar} \delta_{al} \delta_{fr} \delta_{fl} \delta_{el} \delta_{el})^T$ is the control vector. For any control surface $\delta \in \{\delta_{ar}, \delta_{al}, \delta_{fr}, \delta_{fl}, \delta_{el}, \delta_{el}\}$ the aerodynamic forces and moments produced by $\delta$ can be written as:

$$
F_{x2} = \frac{1}{2} \rho V^2 S C_{z2} \delta_{al} = \frac{1}{2} \rho b V^2 S C_{14} \delta_{al}
$$

$$
F_{Ys} = \frac{1}{2} \rho V^2 S C_{zs} M_{\delta} = \frac{1}{2} \rho b V^2 S C_{m3} M_{\delta}
$$

$$
F_{z2} = \frac{1}{2} \rho V^2 S C_{z2} N_{\delta} = \frac{1}{2} \rho b V^2 S C_{n3} N_{\delta}
$$

where $\rho$, $V$, $b$, $c$ and $S$ are the air density, the true airspeed, the wing span, the aerodynamic wing chord and the wing surface respectively, $C_{z2}$, $C_{zs}$, $C_{z2}$, $C_{14}$, $C_{m3}$, $C_{n3}$ are the aerodynamic force and moment coefficients. It is assumed that the aircraft is rigid-body, the weight is constant and the centre of gravity is fixed position. In these conditions, the UAV model can be described with a six degrees of freedom platform subject to the gravity and propulsion forces and to the aerodynamic forces and moments. The UAV’s state vector is $X = (\varphi \theta \alpha \beta \rho q r h)^T$ where $\varphi$ is the bank angle, $\theta$ the pitch angle, $\alpha$ the angle of attack, $\beta$ the sideslip, $p$ the roll, $q$ the pitch, $r$ the yaw and $h$ the height. State space equations proceed from flight mechanics and are detailed in (Bateman et al. [2008]). As it concerns the heading angle and the centre of gravity coordinates, they are not studied here because the FTC problem is an attitude control problem. The general model of the UAV can be written as:

$$
\dot{X} = f(X) + g(X)U
$$

In fault-free mode, the control surfaces are controlled as follows:

- differential deflection of the ailerons $\delta_{al} = -\delta_{ar}$
- symmetric deflection of the flaps $\delta_{fr} = \delta_{fl}$
- symmetric deflection of the elevators $\delta_{el} = \delta_{el}$
- to produce the pitch $\delta_{el}$
- differential deflection of the elevators $\delta_{el} = -\delta_{el}$
- to produce the yaw $\delta_{el}$

Note that flaps are only used to produce a lift increment during takeoff and a drag increment during landing. In fault-free mode, the linearized model of the UAV around an operating point $\{X_{e}, U_{e}\}$ is given by:

$$
\dot{x} = Ax + Bu
$$

The physical flight envelope of this UAV is defined as

$$
X_{P} = \{X \in \mathbb{R}^D : X_{min} \leq X \leq X_{max}\}
$$

For the operating point $X_{e}$, we define the reduced flight envelope $X_{R}$ as the largest ellipsoid contained in the physical flight envelope

$$
X_{R} = \{x \in \mathbb{R}^D : x^T Rx \leq 1\}
$$

Later, this set will be used as a reference set to estimate the domain of attraction of the UAV in closed-loop. A projection onto $\mathbb{R}^2$ of this domain is illustrated on figure 2.

On the other hand, the actuator travels are bounded $U_{min} \leq U \leq U_{max}$. For the $i^{th}$ control $U_i$, the saturation levels are shown on figure 3 and are defined as:

$$
u_{i+} = U_{max} - U_{i_e}$$

$$
u_{i-} = U_{i_{min}} - U_{i_e}$$

With our notations, $u_{i-} \leq 0$ and $u_{i+} \geq 0$. These saturation levels are asymmetric. Later, the method presented in (Hu and Lin [2001]) to estimate the DOA allows only to deal with symmetric saturation levels. Thus, for the $i^{th}$ control, we define the saturation level as

$$
u_{i_{sat}} = \min(u_{i+}, u_{i-})$$

Consequently, the saturation levels are symmetric, therefore the estimation of the DOA will be conservative. Unlike

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1. http://www.u-dynamics.com/aerosim
Hu’s theory in which the saturation levels are chosen between ±1, we have matched their results to consider saturation levels between ±ui_{sat}.

3. ESTIMATION OF DOMAIN OF ATTRACTION

Considering the actuator saturations to estimate the DOA is a problem of paramount importance. After a fault has occurred at time tF and before the nominal controller has been reconfigured to compensate for the effect of the fault at time tR, the system operates under the feedback control designed for normal conditions which provides inappropriate closed-loop control signals. To compensate for severe faults, the healthy actuator control signals may saturate and the system may become uncontrollable. Because of the unstable mode, the state vector moves away from its operating point and may leave the DOA, leading to the loss of the system.

The problem of estimating the DOA for general linear system under saturated linear feedback was presented by Hu in (Hu and Lin [2001]). The main results are recalled below, they are required to understand the design of the FTC presented in section 4. For an operating point {X, U} and under a saturated linear state feedback, the closed loop system is

\[ \dot{x} = Ax + Bsat(Kx) \]  

Where \( x \in \mathbb{R}^n \), \( K \in \mathbb{R}^{m \times n} \) is the state feedback matrix, it has been designed by using an eigenstructure assignment method. Any other method could be used without loss of generality. The standard saturation function is denoted sat, for the \( i^{th} \) control,

\[ sat(u_i) = sign(u_i)min(u_{i_{sat}}, |u_i|) \]  

Hu shows that the saturated state feedback \( sat(Kx) \) can be placed into the convex hull \( co \) of linear feedbacks. Given, the two feedback matrices \( K, H \in \mathbb{R}^{m \times n} \) where \( h_i \) denotes the \( i^{th} \) row of \( H \) and assume that \( [h_i, x] \leq u_{i_{sat}}, i = 1, \ldots m \).

It is shown in (Hu and Lin [2001]) that

\[ sat(Kx) \in co \{ D_iKx + D_i^{-}Hx : i \in [1, 2^n] \} \]  

\( D_i \in \mathbb{R}^{m \times m} \) are matrices whose diagonal elements are either 0 or 1, \( i = 1, 2, \ldots, 2^n \) and \( D_i^{-} = I - D_i \), where \( I \) is the identity matrix. Further, \( \mathcal{L}(K) \) is the region where the feedback control \( sat(Kx) \) is linear in \( x \).

\[ \mathcal{L}(K) := \{ x \in \mathbb{R}^n : [k, x] \leq u_{i_{sat}}, i = 1, \ldots m \} \]  

For \( x_0 = x(0) \in \mathbb{R}^n \) and \( v(t, x_0) \) the state trajectory of the system (11), the DOA of the origin is defined as

\[ \mathcal{S} := \{ x_0 \in \mathbb{R}^n : \lim_{t \to \infty} v(t, x_0) = 0 \} \]  

A measure of the size of this set is obtained with respect to a reference set \( \mathcal{X}_R \). Assume that \( 0 \in \mathcal{X}_R \) is a convex bounded set. For a positive real number, denote

\[ \tilde{\alpha}_R = \{ \tilde{\alpha} : x \in \mathcal{X}_R \} \]  

For a set \( S \subset \mathbb{R}^n \), define the size of \( S \) with respect to \( \mathcal{X}_R \) as

\[ \tilde{\alpha}_R(S) := \sup \{ \tilde{\alpha} > 0 : \tilde{\alpha}_R \subset S \} \]  

For this problem, the reduced flight envelope presented above will play the role of the reference set. Let \( P \in \mathbb{R}^{m \times n} \) be a positive-definite matrix and the ellipsoid

\[ \mathcal{E}(P, \rho) = \{ x \in \mathbb{R}^n : x^T P x \leq \rho \} \]  

Given a feedback matrix \( K \), the optimization problem solved by Hu aims at finding the largest contractive invariant ellipsoid \( \mathcal{E}(P, \rho) \) such that the measure \( \tilde{\alpha}_R(S) \) is maximized. Clearly, if \( \mathcal{E}(P, \rho) \) is contractive and invariant,

![Fig. 3. control in its trim position](image)

then it is inside the DOA. This optimization problem illustrated on figure 4 is given by (Hu and Lin [2001]):

\[ \sup_{P \geq 0, \rho, H} \tilde{\alpha} \]  

s.t.:

a) \( \tilde{\alpha}_R \subset \mathcal{E}(P, \rho) \)

b) \( (A + B(D_iK + D_i^{-}H))^T P + P(A + B(D_iK + D_i^{-}H)) \leq 0 \)

c) \( \mathcal{E}(P, \rho) \subset \mathcal{L}(H) \)

Let

\[ \tilde{\gamma} = \frac{1}{\tilde{\alpha}^2}, Q = \left( \frac{P}{\rho} \right)^{-1}, Z = HQ, \]  

and \( \tilde{\gamma}_i \) the \( i^{th} \) row of \( Z \)

To be solved, this optimization problem is transformed into LMIs (Hu and Lin [2001])

\[ \inf_{Q > 0, Z} \tilde{\gamma} \]  

s.t.:

a) \( \left( \frac{Z^T R I}{I Q} \right) \geq 0 \)

b) \( QA^T + AQ + (D_iKQ + D_i^{-}Z)^T B^T + B(D_iKQ + D_i^{-}Z) < 0 \)

c) \( \left( \frac{z_i^2 Q}{z_i^2 Q} \right) \geq 0 \)

Let \( \tilde{\gamma}^* \) the optimum of this problem with the solutions \( Q^* \) and \( Z^* \), then \( \tilde{\gamma}^* = \frac{1}{\sqrt{\tilde{\gamma}^*}} \).

Here, and without loss of generality, \( \rho \) was chosen equal to 1.

4. FAULT TOLERANT CONTROL STRATEGY

When \( k \) control surfaces are stuck, the equilibrium of forces and moments is broken and the UAV state vector

![Fig. 4. Estimate of DOA](image)
The proposed FTC strategy consists of two stages. Firstly, a new operating point \( \{X_f^h, U_c^h\} \) is calculated, nonlinear characteristics of the UAV, state variable limitations and control saturations are taken into account. To compute this new operating point, it is assumed that the faulty control and their positions are known. Furthermore, the usual deflection constraints of the healthy control surfaces are released (3) and each one of the \( 7-k \) healthy actuators is trimmed separately. Secondly, for this new operating point, a pre-computed linear state feedback controller is implemented. FTC objectives aim at maximizing the DOA while ensuring a fast and soft transient toward the new equilibrium. The design of the FTC linear state feedback controller requires a heavy computational load. Moreover a controller must be theoretically computed for each control surface fault position. For these reasons the controllers are pre-computed. The computation of an operating point in faulty mode was processed in (Bateman et al. [2008]) and is briefly recalled. Here, we focus on the design of the feedback controller.

### 4.1 Operating point computation in faulty mode

In (Bateman et al. [2008]), an optimization method based on a sequential programing quadratic algorithm was presented to compute in real time a new operating point. It consists in trimming the healthy controls in order to:

- satisfying equation \( f(X) + g(X)U = 0 \) (22)
- finding solutions included in the physical flight domain and in the control variation ranges,
- keeping the new operating point close to the fault-free operating point,
- minimizing the control deflections required to reach the new trims.

For this new operating point \( \{X_f^h, U_c^h\} \), the faulty linearized model writes:

\[
\dot{x} = A_f x + B_f u^h
\]

Note that if no operating point satisfying an equilibrium exists, the aircraft will be lost. In this case, degraded modes of operation such as a descent at minimum kinetic energy may be considered. It is also worth to notice that the new trims determine new saturation levels.

### 4.2 Linear state feedback controller design

The FTC controller is a saturated state feedback such that \( u^h = sat(Fx) \) with \( F \in \mathbb{R}^{n \times 7-k} \). It is designed in order to maximize the DOA while the poles are placed in an LMI region as illustrated on figure 5. In faulty mode, this strategy aims at increasing the chances of saving the UAV while the current state vector is steered toward the new equilibrium with a damping factor greater than or equal to \( \hat{\alpha} \) and a time response less than or equal to \( \frac{1}{\kappa} \).

Firstly, it is necessary to compute the feedback matrix \( H \) such that the estimate of the DOA is maximized with respect to a reference set. This matrix is obtained by solving the optimization problem (24) (Hu and Lin [2001]). This problem is similar to (19) with an extra optimization parameter \( M \).

\[
\sup_{P \succ 0, \rho, \kappa, M, H} \hat{\alpha} \quad \text{s.t.} \quad \begin{align*}
& a) \hat{\alpha}X_R \subset \mathcal{E}(P, \rho) \\
& b) (A_f + B_f(D_c M + D_i^- H))^T P + P (A_f + B_f(D_c M + D_i^- H)) < 0 \quad i \in [1, 2^m] \\
& c) \mathcal{E}(P, \rho) \subset \mathcal{L}(H)
\end{align*}
\]

This optimization problem is transformed into an LMI problem. Changing the variables (see 20) leads to

\[
\inf_{Q \succ 0, Z, \gamma} \tilde{\gamma} \quad \text{s.t.} \quad \begin{align*}
& a) (\tilde{\gamma} R I_d + I_d Q)^T \geq 0 \\
& b) QA_f^T + A_f Q + (D_i^- H^* Q + D_i Z)^T B_f^T + B_f (D_c M Q + D_i^- Z) < 0 \quad i \in [1, 2^m] \\
& c) (\tilde{\gamma} z_{i+1}^T Q)^T \geq 0 \quad i = 1, \ldots, m
\end{align*}
\]

\[\text{Let } H^* = Z^* Q^{-1} \text{ the optimal feedback matrix which maximizes the estimate of the DOA of the system.}\]

The LMI formulation proposed hereafter can be seen as an extension to Hu’s works. \( \hat{\gamma} \) is set equal to the optimal value of (25). It follows that it guarantees a known DOA and which satisfies dynamic performances. Here, controlling the damping and the decay rate. Pole placement in the LMI regions can be formulated as an LMI optimization problem (26.d), (26.e), (26.f) (Chilali et al. [1999]).
the fault diagnostic system. When a fault occurs, a new operating point and a feedback matrix must be computed for each fault situation. The time required to compute the feedback matrix is incompatible with the speed of the decisions which have to be made. Thus, feedback matrices for the FTC system are pre-computed and for each fault situation a controller is chosen in a bank of controllers. In order to reduce the number of controllers, for each control surface, the range of fault positions is splitted among sectors. In each sector a unique controller must satisfy the LMI region constraints and must guarantee a measure of the DOA \( \tilde{\alpha}_R \geq 1 \). These last conditions require the DOA to be greater than or equal to the reduced flight envelope.

5. RESULTS OF SIMULATIONS

The UAV flies level, its operating point is \( \phi_e = \beta_e = 0^\circ \), \( \theta_e = \alpha_e = 4^\circ \), \( V_e = 25m/s \), \( p_e = q_e = r_e = 0^\circ/s \), \( h_e = 200m \). It is obtained with the trims \( \delta_{\alpha_e} = 0.57 \), \( \delta_{ar_e} = \delta_{fl_e} = 0^\circ \), \( \delta_{a_r} = \delta_{sl} = -3.9^\circ \).

The physical flight envelope \( X_R \) is \(-45^\circ \leq \phi \leq 45^\circ \), \(-15^\circ \leq \theta \leq 15^\circ \), \( 15m/s \leq V \leq 50m/s \), \(-6^\circ \leq \alpha \leq 17^\circ \), \(-6^\circ \leq \beta \leq 17^\circ \), \(-90^\circ s^{-1} \leq p \leq 90^\circ s^{-1} \), \(-90^\circ s^{-1} \leq q \leq 90^\circ s^{-1} \), \(-90^\circ s^{-1} \leq r \leq 90^\circ s^{-1} \), \( 0m \leq h \leq 3000m \). In these conditions, the reference set \( X_R \) which is also the reduced flight envelope is given by (6), (7). For this flight stage, the nominal controller is designed with an eigenstructure assignment method. The map of poles is illustrated on the figure 6. For this setting, the size of the DOA with respect to the reference set \( X_R \) is obtained by solving (20) and is equal to \( \tilde{\alpha}^* = 1.1 \).

The right elevator locks in position \( \delta_{cr} = 0^\circ \) at time \( t_F = 40s \). With the nominal operating point and the nominal controller, the problem (20) has no solution and the UAV is unstable.

In this faulty situation, an equilibrium exists with the following operating point: \( \phi_f = \beta_f = 0^\circ \), \( \theta_f = \alpha_f = 3^\circ \), \( V_f = 25m/s \), \( p_f = q_f = r_f = 0^\circ/s \), \( h_f = 200m \) obtained with the trims \( \delta_{\alpha_f} = 0.86 \), \( \delta_{ar_f} = -0.16^\circ \), \( \delta_{sl_f} = -0.69^\circ \), \( \delta_{hr} = 7.8^\circ \), \( \delta_{fl_f} = 10.1^\circ \), \( \delta_{hl} = 0^\circ \). This new operating point imposes a new reference set \( X_R \) and new saturation levels. For this operating point the state and the control matrix of the linearized model are \( A_f \) and \( B_f \).

The feedback matrix \( H \) maximizes the DOA of the saturated linear state feedback system

\[
\dot{x} = A_f x + B_f sat(Hx)
\]  

The size of its estimation with respect to the new \( X_R \) is given by (25) and is equal to \( \alpha^* = 83.46 \).

In faulty mode, a damping ratio greater than 0.5 is desired, so \( q = 60^\circ \), and for the slowest mode, a decay time less than 10s so \( \omega_1 = -0.3 \). As for \( \omega_2 \), it is chosen equal to -30. This choice aims at keeping the closed loop poles in the neighborhood of the open-loop poles. The feedback matrix \( F \), which maximizes the DOA of the saturated linear state feedback

\[
\dot{x} = A_f x + B_f sat(Fx)
\]  

while it guarantees the poles in the desired LMI region, is given by (26). The size of the estimation of the DOA is \( \alpha^* = 83 \) with respect to the new reference set \( X_R \). Here, the estimation of the DOA is greater than the reduced flight envelope. This means that the physical domain determines the critical limit of use of the aircraft. Moreover, it seems surprising that the estimation of the DOA in faulty mode is greater than the estimation of the DOA in fault-free mode. In fact the fault-free aircraft has only 5 controls, the throttle, the 2 ailerons and the 2 elevators. Moreover, these control surfaces are constrained (3). In faulty mode, the constraints (3) are released and the throttle plus the 6 healthy controls are used (including the flaps).

Results of simulations are illustrated on figure 7 and 8. The right elevator locks in position \( \delta_{cr} = 0^\circ \) at time \( t_F = 40s \). It appears that, without the FTC strategy, the height decreases and the UAV is lost. The FTC controller is triggered at time \( t_R = 42s \), the state variables are steered toward their equilibrium in less than 10 seconds with a good damping ratio.

![Fig. 5. The LMI region](image)

![Fig. 6. The map of poles](image)

6. CONCLUSIONS AND PERSPECTIVES

In this paper, a fault-tolerant control strategy for a UAV where control surfaces are stuck, had been designed. The design of the fault tolerant controller takes explicitly into account the physical limits of the system and aims at maximizing the domain of attraction while it guarantees the dynamic performances. Future works will deepen the understanding of the maximum time duration for fault diagnostic and accommodation. Indeed, few works deal with the problem of the maximum duration allowed between fault occurrence and fault reconfiguration (Zhang and Jiang (2006)). After fault occurrence time \( t_F \) and
until reconfiguration time $t_R$, the UAV operates under the feedback controller designed for nominal conditions and the estimation of its DOA in faulty mode $E(P_F, \hat{P}_F)$ is obtained by solving (21) (see on figure 9). At reconfiguration time $t_R$, a new controller is switched on. The feedback matrix and the estimation of the DOA in reconfigured mode are obtained by solving (26). Let $E(P_R, \hat{P}_R)$ this set.

When a fault occurs, the state vector moves in the initial DOA (which estimation is $E(P_F, \hat{P}_F)$) until the border of this set (state trajectory drawn with a solid line). If it crosses this border, the state vector moves with a dynamic defined by the first order unstable mode in the direction of the eigenvector associated with this mode (dashed line). On figure 9 the state vector at $X(t_R)$ is inside $E(P_R, \hat{P}_R)$ AND the physical domain $X_F$. Therefore, the state vector is in the stable region and the fault tolerant controller steers it toward the new operating point $X_f^f$ (dotted line). Maximum duration allowed for $t_R - t_F$ requires the computation of the minimum distance between the border of $E(P_F, \hat{P}_F)$ and the ones of $X_F$ OR $E(P_R, \hat{P}_R)$ in the direction of the eigenvectors associated with the unstable poles.

**Fig. 9.** The state trajectory in faulty situation and the various sets

**REFERENCES**


